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Sampling and quantization problems for distributed detection problems are investigated. Sampling algorithms for weak signal detection problems are derived. Sampling and quantization issues under communication constraints are treated. A new paradigm for distributed detection is presented.

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FINAL REPORT

DISTRIBUTED DETECTION THEORY AND DATA FUSION

(Grant No. F49620-94-0182)

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Distributed signal detection systems have been shown to be quite effective due to their advantages that include higher reliability and survivability, enhanced system performance and shorter processing time. These systems consist of multiple remotely located sensors that observe a common phenomenon. Some data processing is carried out at these peripheral detectors and processed information is sent to a central detector for fusion. When observations at peripheral detectors are continuous-time, they need to be sampled. One important issue is the design of sampling schemes at the peripheral sensors. An option is to employ uniform sampling but it is not necessarily optimum. Also, sampling schemes designed to ensure signal reconstruction with minimal loss due to sampling may not be the best when used in signal detection systems. We considered the problem of sampling design for Gaussian signal detection problems. Due to the analytical intractability of the probability of error criterion, our approach was based on the class of Ali-Silvey distance measures. Sampling points were determined that maximized the Ali-Silvey distance measure between the class conditional densities. Specifically, the Bhattacharyya distance, the I-divergence, the J-divergence and the Chernoff distance were used. The known signal case and the random signal case under strong signal assumptions were considered. This methodology was extended for the weak signal case. A summary of these results is included in Appendix A. In the context of distributed signal detection both sampling and quantization are carried out at the peripheral detectors. Their joint design when there is a constraint on the communication bandwidth of the outgoing link was investigated. These results are available in the Ph.D. dissertation by C.T. Yu. The issue of constrained communication bandwidth in distributed detection is further considered by proposing a different paradigm for distributed detection. This approach combines the features of both centralized and hard decision decentralized detection problems. An extended abstract that outlines the results is provided in Appendix B. Publications stemming from AFOSR sponsorship during this research period are given next.

Journal Papers

T.C. WANG and P.K. VARSHNEY, "Measurement Preprocessing Approach for Target Tracking in a Cluttered Environment," *IEE Proceedings, Part F,* vol. 141, pp. 151-158, June 1994.

S. ALHAKEEM and P.K. VARSHNEY, "A Unified Approach to the Design of Decentralized Detection System," *IEEE Transactions in Aerospace and Electronic Systems*, pp. 9-20, Jan. 1995.

C. TUMULURI and P.K. VARSHNEY, "An Evidential Extension of the MRII Training Algorithm for Detecting Erroneous MADALINE Responses," *IEEE Transactions on Neural Networks*, pp. 880-892, July 1995.

J. MICHELS, P.K. VARSHNEY and D.D. WEINER, "Multichannel Detection Using a Model-Based Approach," *IEEE Transactions in Aerospace and Electronic Systems,* July 1995.

Conference Papers

C.T. YU and P.K. VARSHNEY, "Sampling Design for Weak Signal Detection Problems," Proc. of the 1994 Conference on Information Sciences and Systems, Princeton, NJ, March 1994.

T. TSAO, P.K. VARSHNEY and D.D. WEINER, "Discrete Wigner-Ville Distributed Based Radar Receivers," Proc. of the 1994 Conference on Information Sciences and Systems, Princeton, NJ, March 1994.

M.K. UNER and P.K. VARSHNEY, "CFAR Processing in Nonhomogeneous Clutter, Proc. of MELECON '94, Antalya, Turkey, April 1994.

C. TUMULURI and P.K. VARSHNEY, "An Evidential Extension of the MRII Training Algorithm for Detecting Erroneous MADALINE Responses," Proc. of the Int. Conf. on Info. Processing and Management of Uncertainty in Knowledge-Based Systems, Paris, France, July 1994.

C.T. YU and P.K. VARSHNEY, "Sampling and Quantization Issues in Distributed Detection," Joint Conf. on Information Sciences, Pinehurst, NC, Nov. 1994.

C.T. YU and P.K. VARSHNEY, "Sampling Theory for Hypothesis Testing Problems," Presented at the 1995 Canadian Workshop on Information Theory, Lac Delage, Quebec, May 1995.

Ph.D. Dissertations

T. Tsao, "Radar Signal Detection and Estimation Using Time-Frequency Distribution," December 1994.

C.T. Yu, "Sampling and Quantizer Design for Hypothesis Testing Problems," December 1994.

APPENDIX A

Summary

Recently, sampling design for detection problems has been considered in [1, 2, 3]. In[1], random and deterministic sampling schemes for the detection of known signals in Gaussian noise were considered. Use of the decision statistic based on sampled data in the receiver, instead of the continuous-time detector (matched filter/correlation receiver), naturally results in a degradation of performance. Deterministic and random sampling schemes were designed so as to minimize the degradation in the performance of the detector. The performance measure for the detectors was based on the so called generalized signal-to-noise ratio. In[2], an asymptotically optimal periodic sampling design for the Gaussian hypothesis testing problem was presented. Under very general conditions, the probabilities of error (Type I and Type II errors) decrease exponentially with increasing sample size. For this problem, an asymptotically optimal periodic sampling scheme was designed by maximizing the rate of exponential decrease. In[3], the sampling scheme for Gaussian hypothesis testing problem was obtained so as to maximize the Ali-Silvey distances between the class conditional densities. All of the above sampling procedures[1, 2, 3] are applicable to the strong signal case. They do not treat the weak signal detection problem. In this paper, we consider sampling design for the Gaussian detection problem in the weak signal case.

In many practical situations, we are interested in the weak signal detection problem which can be formulated as a hypothesis testing problem for testing hypothesis H_0 versus an alternative hypothesis H_1

$$H_1 : r(t) = \theta s(t) + n(t) \qquad a \leq t \leq b$$

$$H_0 : r(t) = n(t) \qquad a \leq t \leq b$$

where $r(t)$ is the observation over the interval $[a, b]$, $n(t)$ is a Gaussian random process with zero mean and covariance function $R_n(t, \tau)$, and $s_i(t), i = 0, 1$, is the signal. The signal could be a deterministic signal or a Gaussian random process with mean $m_{s_i}(t)$ and covariance function $R_{s_i}(t, \tau)$, i=0, 1. s(t) is assumed to be independent of the noise, $n(t)$.

θ is a parameter with small positive real value so that $\theta s(t)$ represents the weak signal as θ approaches zero. Without loss of generality, the average power of $s(t)$ and that of $n(t)$ are normalized to unity so that θ can be used to control the signal to noise ratio. This is a well-known model for the weak signal detection problem which has been studied extensively in the literature. For example, Middleton[4] derived the locally optimum detector (LOD) by expanding the likelihood ratio test in terms of a power series and truncating to a first order approximation. In the limit as the signal tended to zero, the canonical structure of the LOD was established for the weak signal detection problem. Kassam [5] analyzed the performance of the LOD when the sample size approaches infinity. Here we consider the design of sampling schemes for weak signal detection.

Let $\mathbf{r}$, $\mathbf{n}$ and $\theta\mathbf{s}$ represent the sampled observation, noise and signal data vectors respectively. Also, let $p(\mathbf{r}|H_1)$, $p(\mathbf{r}|H_0)$, and $f_{\mathbf{N}}(\mathbf{n})$ denote the joint probability density functions (PDFs) of $\mathbf{r}$ under H_1, of $\mathbf{r}$ under H_0 and of noise respectively. The class of Ali-Silvey distance measures[6, 7] between the conditional densities, $p(\mathbf{r}|H_1)$ and $p(\mathbf{r}|H_0)$, is defined by the general expression, $D(p(\mathbf{r}|H_1), p(\mathbf{r}|H_0)) = f(E_0[C(L)])$, where f is an increasing function; E_0 denotes expectation with respect to p_0; C is a convex function; L is the likelihood ratio defined as $\frac{p(\mathbf{r}|H_1)}{p(\mathbf{r}|H_0)}$.We note that the likelihood ratio approaches unity as signal becomes very weak (i.e., as θ approaches zero). This is due to the fact that

$$p(\mathbf{r}|H_1) \approx p(\mathbf{r}|H_0) = f_{\mathbf{N}}(\mathbf{n}) \qquad as \quad \theta \to 0.$$

Also, the Ali-Silvey distance measure between the conditional densities, $p(\mathbf{r}|H_1)$ and $p(\mathbf{r}|H_0)$, goes to 0 as θ goes to 0, i.e.,

$$\lim_{\theta \to 0} D(p(\mathbf{r}|H_1), p(\mathbf{r}|H_0)) = \lim_{\theta \to 0} f(E_0[C(L)]) = 0.$$

Therefore, in this weak signal case, the sampling design criteria developed in[3], i.e., the Ali-Silvey distance measures between $p(\mathbf{r}|H_1)$ and $p(\mathbf{r}|H_0)$, degenerate. To overcome this difficulty, a series expansion approach will be employed. The MacLaurin series expansion

of the Ali-Silvey distance measure, $D(p(\mathbf{r}|H_1),\ p(\mathbf{r}|H_0))$, about the parameter θ=0 can be written as

$$D(p(\mathbf{r}|H_1),\ p(\mathbf{r}|H_0)) = \sum_{i=0}^{\infty}[\frac{1}{i!}(\frac{\partial^i D(p(\mathbf{r}|H_1),\ p(\mathbf{r}|H_0))}{\partial\theta^i})|_{\theta=0}]\cdot\theta^i.$$

Because of the fact $\frac{\partial^0 D(p(\mathbf{r}|H_1),\ p(\mathbf{r}|H_0))}{\partial\theta^0}|_{\theta=0} = D(p(\mathbf{r}|H_1), p(\mathbf{r}|H_0))|_{\theta=0} = 0$, the series expansion of $D(p(\mathbf{r}|H_1), p(\mathbf{r}|H_0))$ becomes

$$D(p(\mathbf{r}|H_1),\ p(\mathbf{r}|H_0)) = \sum_{i=1}^{\infty}[\frac{1}{i!}(\frac{\partial^i D(p(\mathbf{r}|H_1),\ p(\mathbf{r}|H_0))}{\partial\theta^i})|_{\theta=0}]\cdot\theta^i.$$

As discussed above, in the limit as the signal strength tends to zero (i.e., $\theta \to 0$), the distance measure between $p(\mathbf{r}|H_1)$ and $p(\mathbf{r}|H_0)$ becomes approximately equal to 0. Therefore, instead of maximizing the distance measures $D(p(\mathbf{r}|H_1),\ p(\mathbf{r}|H_0))$, we can maximize the first nonzero coefficient of the series expansion of $D(p(\mathbf{r}|H_1),\ p(\mathbf{r}|H_0))$ to design the sampling schemes. This is because the first non-zero term is the dominant term in the series expansion of the distance measures due to the fact that $\theta << 1$. Hence, the first nonzero coefficient of the MacLaurin series expansion of $D(p(\mathbf{r}|H_1), p(\mathbf{r}|H_0))$ will be derived. In order to determine this quantity, we consider the following two cases separately: one is when the signal is deterministic and the other is when the signal is a Gaussian random process.

A. The Deterministic Signal Case

In this case, the signal is deterministic, and the noise is a Gaussian random process with zero mean and a known autocorrelation function, $R_n(t,\tau)$, where $R_n(t,t)$ is set to unity without loss of generality. The joint PDFs of the sampled data, $\mathbf{r}^T = [r(t_1)\ r(t_2)\cdots r(t_N)]$, under hypotheses H_0 and H_1 are N-dimensional Gaussian density functions with mean vector and covariance matrix, $(\mathbf{0}, \mathbf{K_n})$ and $(\theta\mathbf{s}, \mathbf{K_n})$, respectively, i.e.,

$$p(\mathbf{r}|H_0) = \frac{1}{(2\pi)^{N/2}|\mathbf{K_n}|^{1/2}}\cdot\exp(-\frac{1}{2}\mathbf{r}^T\mathbf{K_n}^{-1}\mathbf{r})$$

$$p(\mathbf{r}|H_1) = \frac{1}{(2\pi)^{N/2}|\mathbf{K_n}|^{1/2}}\cdot\exp(-\frac{1}{2}(\mathbf{r}-\theta\mathbf{s})^T\mathbf{K_n}^{-1}(\mathbf{r}-\theta\mathbf{s})).$$

It can be shown[8] that the first non-zero coefficient of the series expansion of $D(p(\mathbf{r}|H_1), p(\mathbf{r}|H_0))$ is

$$\frac{\partial^2 D}{\partial \theta^2}|_{\theta=0} = C_1 \cdot E_0[(\frac{\partial L}{\partial \theta}|_{\theta=0})^2] \tag{1}$$

where D denotes $D(p(\mathbf{r}|H_1), p(\mathbf{r}|H_0))$, C_1 is a constant that depends on the functions f and C used to define the specific distance measure in the class of Ali-Silvey distance measure. For example, C_1=1, 2, and $\frac{1}{4}$ for I-divergence, J-divergence and Bhattacharyya distance defined in[7, 9]respectively. For this Gaussian noise case, it has been shown[8] that

$$\frac{\partial^2 D}{\partial \theta^2}|_{\theta=0} = C_1 \cdot \mathbf{s}^T \mathbf{K_n}^{-1} \mathbf{s}.$$

Since the constant C_1 is independent of the sampling points, t_1 to t_N, we conclude that the sampling points are obtained so as to maximize $\mathbf{s}^T \mathbf{K_n}^{-1} \mathbf{s}$ for the weak signal detection problem in the known signal case. Sampling points are independent of the choice of distance measures in the class of Ali-Silvey distance measures. It is interesting to note that this quantity is identical to that obtained in[3] for the strong signal case when the constant is ignored. This is not surprising in view of the fact that the statistic of the likelihood ratio test for the known signal Gaussian problem[10] is

$$LR(\mathbf{r}) = \mathbf{s}^T \mathbf{K_n}^{-1} \mathbf{r}$$

and the locally optimum detector (LOD) statistic[5] is

$$L_{LOD}^D = LOD(\mathbf{r}) = \mathbf{s}^T \mathbf{K_n}^{-1} \mathbf{r}. \tag{2}$$

Hence, for the known signal Gaussian detection problem, the sampling design criterion based on the distance measures for the weak and strong signal cases are identical.

Let the objective function in Equation (1) be written as

$$D_{YV} = C_1 \cdot E_0[(\frac{\partial L}{\partial \theta}|_{\theta=0})^2] = C_1 \cdot E_0[(L_{LOD}^D)^2] = f(E_0[C(L_{LOD}^D)]) \tag{3}$$

where the functions are defined as $f(x) = C_1 \cdot x$, $C(x) = x^2$, andL^D_{LOD} is the LOD statistic in the deterministic signal case shown in Equation(2). Since f is an increasing function, and C is a convex function, interestingly enough, D_{YV} represents a new distance measure belongs to in the class of Ali-Silvey distance measures.

B. The Random Signal Case

In this case, $s(t)$ and $n(t)$ defined earlier are Gaussian random processes with mean and covariance functions, $(s_m(t),\ R_s(t,\tau))$ and $(0,\ R_n(t,\tau))$, respectively, where $R_s(t,t)$ and $R_n(t,t)$ are set to 1. Then, the joint PDF of the N samples of observation, $\mathbf{r}^T = [r(t_1)\ r(t_2)\cdots r(t_N)]$, under hypothesis H_0 is a N-dimensional Gaussian density function with zero mean and covariance matrix, $\mathbf{K_n}$. Let the PDF of N samples of $s(t)$, $\mathbf{s}^T = [s(t_1)\ s(t_2)\cdots s(t_N)]$, be denoted as $f_{\mathbf{S}}(\mathbf{s})$ with a N-variate normal density function, $\mathcal{N}(\mathbf{s_m}, \mathbf{K_s})$. The noise is assumed to be independent of the signal. Thus, the PDF of $\mathbf{r}$ under hypothesis H_1 can be written as

$$p(\mathbf{r}|H_1) = \int_{\mathbf{s}} f_N(\mathbf{r} - \theta\mathbf{s}) \cdot f_{\mathbf{S}}(\mathbf{s}) d\mathbf{s} = E_s[f_N(\mathbf{r} - \theta\mathbf{s})] \tag{4}$$

where $f_N(\mathbf{n}) = f_N(\mathbf{r}|H_0) = p(\mathbf{r}|H_0)$, and E_s denotes expectation with respect to $f_{\mathbf{S}}(\mathbf{s})$.

When the mean vector, $\mathbf{s_m}$, is non-zero, the first nonzero coefficient of the series expansion of $D(p(\mathbf{r}|H_1),\ p(\mathbf{r}|H_0))$[8] is

$$\begin{aligned} \frac{\partial^2 D}{\partial \theta^2}|_{\theta=0} &= C_1 \cdot E_0[(\frac{\partial L}{\partial \theta}|_{\theta=0})^2] \\ &= C_1 \cdot \mathbf{s_m}^T \mathbf{K_n}^{-1} \mathbf{s_m}. \end{aligned}$$

Hence, in this situation, all the results are essentially the same as in the deterministic signal case when $\mathbf{s}$ is replaced by $\mathbf{s_m}$. Therefore, we consider the more interesting case when $\mathbf{s_m} = \mathbf{0}$. Due to the fact that $\mathbf{s_m} = \mathbf{0}$, the first nonzero coefficient of the series expansion of $D(p(\mathbf{r}|H_1),\ p(\mathbf{r}|H_0))$[8] becomes

$$\frac{\partial^4 D}{\partial \theta^4}|_{\theta=0} = C_2 \cdot E_0[(\frac{\partial^2 L}{\partial \theta^2}|_{\theta=0})^2] \tag{5}$$

where C_2 is a constant which is related to the choice of the functions, f and C as indicated earlier. Note that the results obtained so far are valid for the signal vector $\mathbf{s}$ with any arbitrary N-variate density function. We now let $s(t)$ to be a zero-mean Gaussian random process. Then the sampled data vector, $\mathbf{s}$, has an N-dimensional joint Gaussian density function denoted as $\mathcal{N}(\mathbf{0}, \mathbf{K_s})$. In this situation, Equation (5) can be evaluated to yield[8]

$$\frac{\partial^4 D}{\partial \theta^4}|_{\theta=0} = 2C_2 \cdot tr[(\mathbf{K_s}\mathbf{K_n}^{-1})^2] \tag{6}$$

The result obtained in Equation (6) depends on the covariance matrices, $\mathbf{K_s}$ and $\mathbf{K_n}$, which come from the covariance functions, $R_s(t,\tau)$ and $R_n(t,\tau)$, through the sampling points, $t_1, t_2, \cdots, t_N$. Therefore, the first non-zero coefficient of the series expansion of D depends only on the sampling points and the covariance functions. As before, we rewrite Equation (5) as

$$D_{YV} = C_2 \cdot E_0[(\frac{\partial^2 L}{\partial \theta^2}|_{\theta=0})^2] = C_2 \cdot E_0[(L_{LOD}^R)^2] = f(E_0[C(L_{LOD}^R)]) \tag{7}$$

where again $f(x) = C_2 \cdot x$, $C(x) = x^2$, and L_{LOD}^R is the LOD statistic in the random signal with zero mean case. In this case, again the first non-zero coefficient of the series expansion of D yields a new distance measure denoted as D_{YV} that belongs to the class of Ali-Silvey distance measures.

Thus, a new distance measure for the weak signal Gaussian detection problem is developed in this paper. This distance measure is obtained by expanding the Ali-Silvey distance between the class conditional densities in a power series and then considering the first non-zero coefficient. Sampling points that maximize the new distance measure are obtained by employing the iterative algorithm suggested in[3]. A numerical example is presented for illustration. In the example, we compare the detection performance of our sampling design based on the new distance measure and that of the uniform sampling scheme. We demonstrate that the sampling design based on the new distance measure outperforms the uniform sampling scheme.

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APPENDIX B

Extended Abstract

Recently, the design and analysis of distributed sensor networks for signal detection and estimation (decentralized detection/estimation systems) have attracted substantial interest in the literature. This is because decentralized systems generally have the advantages of higher reliability, survivability, and shorter decision time over centralized systems. A decentralized detection system usually contains a number of remotely located local sensors that observe a common phenomenon and a data fusion center(or global decision maker) that makes a final decision. The local sensors are linked to the data fusion center by transmission channels. If there are no constraints on the transmission channels, all the raw data (or local likelihood ratio) at local sensors can be transmitted to the global decision maker for data processing. In this case, signal processing becomes centralized in nature and conventional optimal procedures can be implemented at the fusion center. In many practical situations, there are limitations on the transmission channels. Also, local sensors are provided with processing capabilities. In this case, local sensors pre-process their observations individually and convey the compressed version of sensor data to the fusion center where the received information is appropriately combined to make the final decision. Decentralized detection systems have been designed using various approaches such as the Bayesian approach, the Neyman-Pearson approach, the min-max criterion and the Ali-Silvey distance criterion, e.g., [1, 2, 3, 4]. Tenney and Sandell[1] considered a distributed detection system with a fixed fusion rule. Chair and Varshney[2] developed an optimal fusion rule with fixed local detectors. Hoballah and Varshney[3] presented a generalized Bayesian design approach for a decentralized detection system.

In this paper, we present a novel paradigm for the decentralized detection problem under communication constraints. The proposed approach is flexible and combines the features of both centralized and hard decision decentralized detection problems. Under specified constraints, we design the optimum decentralized detection scheme. The system can operate

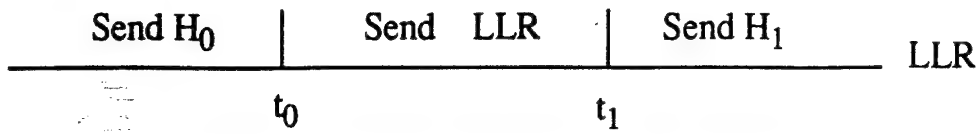

Figure 1: Decision scheme at a local sensor

at the two extremes, i.e., it can be a centralized system or a hard decision decentralized detection system, or anywhere in-between. In this scheme, local sensors send a binary (hard) decision to the fusion center when the local sensors have a higher confidence in the decision, otherwise a perfect version of the LLR (in practice, a finely quantized version of the LLR) is sent. The degree of confidence at which this switch is made is determined by the specified communication constraint. The fusion center makes a final decision based on the received information from local sensors. A local sensor transmits the LLR to the fusion center if the LLR falls in the region, $t_0 \leq LLR \leq t_1$, otherwise it sends a binary decision as shown in Figure 1. This scheme provides a tradeoff between the average communication rate and the resulting system performance. One may determine the average communication rate to attain the desired system performance or one may determine the achievable system performance under a given constraint on the average communication rate.

Problem Formulation

The decentralized detection system to be considered is shown in Figure 2. Observation samples at the local sensors are denoted by $\mathbf{r_i}$, $i = 1, \cdots, M$, and their joint conditional densities are assumed known. There is no communication among local sensors. Based on its own observation $\mathbf{r_i}$, each local sensor makes a local decision $u_i \in \{0,\ 1,\ 2\}$, $i = 1, \cdots, M$, where $u_i = 0$ and $u_i = 1$ represent the fact that the i^{th} local sensor decides hypotheses H_0 and H_1 and correspondingly sends a zero and a one to the fusion center. $u_i = 2$ indicates that the i^{th} local sensor computes and sends its LLR L_i to the fusion center. Let u_{F_i} represent the output of the sensor i, i.e., $u_{F_i} = u_i$ when u_i=0 or 1; $u_{F_i} = L_i$ when u_i=2. Local sensor outputs are transmitted to the fusion center where a global decision is made based on the received data vector, $\mathbf{u_F}^T$=$[u_{F_1}\ u_{F_2}\ \cdots\ u_{F_M}]$.

In this decentralized detection problem, the idea is to partition the LLR (observation space) at each sensor into three disjoint regions $\Re_0^{(i)}$, $\Re_1^{(i)}$ and $\Re_2^{(i)}$, $i = 1, \cdots, M$, and assign the corresponding value to u_i, i.e.,

$$u_i = \begin{cases} 0, & L_i(\mathbf{r_i}) \in \Re_0^{(i)}, \\ 1, & L_i(\mathbf{r_i}) \in \Re_1^{(i)}, \\ 2, & L_i(\mathbf{r_i}) \in \Re_2^{(i)} = \Re^{(i)} - \Re_1^{(i)} - \Re_0^{(i)}, \end{cases}$$

where $\mathbf{r_i}$ is the sampled observation vector and $L_i(\mathbf{r_i})$ is the local likelihood ratio (LLR) at the i^{th} sensor. When $\Re_2^{(i)}$, $i = 1, \cdots, M$, are null sets, the local sensors make a hard decision only and this kind of problem has been studied in [1, 3]. On the other hand, when $\Re_0^{(i)}$ and $\Re_1^{(i)}$, $i = 1, \cdots, M$, are null, the problem reduces to a centralized detection problem. In this case, conventional detection procedures can be implemented. Thus, the decentralized detection problem under consideration here is an intermediate problem between the hard decision decentralized detection problem and the conventional centralized detection problem. The probability that $L_i(\mathbf{r_i})$ is transmitted from the sensor i is employed as a measure of the transmission rate on the channel i. We define

$$R_i = p(\text{send } L_i) = 1 - p(\text{send } H_0 \text{ or } H_1), \quad i = 1, \cdots, M. \tag{1}$$

Note that R_i=1, $i = 1, \cdots, M$, represents the centralized case, and R_i=0, $i = 1, \cdots M$, represents the case that hard decisions are made at the local sensors. We are interested in examining the flexible hybrid decision scheme in the decentralized detection system with a lower average communication rate (as compared to the centralized detection problem) on the channels linking local sensors to the fusion center.

System Design

In the Bayesian hypothesis testing problem, the optimal local decision rules and the fusion rule are obtained so as to minimize the average cost. The Bayes risk function to be minimized can be written in the form

$$R_{risk} = \sum_{i=0}^{1} \sum_{j=0}^{1} C_{ij} \pi_j p(\text{Decide } H_i \mid H_j \text{ is true}), \tag{2}$$

where C_{ij} is the cost of deciding H_i when H_j is true and π_j is the a priori probability of H_j, $i,\ j = 0,\ 1$.

Design of a decentralized detection system involves specifying both the local decision rules and the global decision rule. By employing the person-by-person optimization methodology, the system is designed so as to minimize the risk function. The system is specified by

- Optimal local decision rule at sensor k, $k = 1, \cdots, m$:

$$u_k = \begin{cases} 0, & \frac{p(\mathbf{r_k}|H_1)}{p(\mathbf{r_k}|H_0)} < t_{20}^{(k)}, \\ 1, & \frac{p(\mathbf{r_k}|H_1)}{p(\mathbf{r_k}|H_0)} > t_{12}^{(k)}, \\ 2, & \text{otherwise.} \end{cases} \tag{3}$$

- Optimal fusion rule:

$$\frac{p(\mathbf{u_F^*}|H_1)}{p(\mathbf{u_F^*}|H_0)} \underset{u_0=0}{\overset{u_0=1}{\gtrless}} \frac{C_f}{C_d}, \tag{4}$$

where u_F^* is the one of the 3^M possible combinations of u_F.

Motivated by the difficulty and excessive computational requirements of the above PBPO system design, a simplified design procedure based on the class of Ali-Silvey distance measures is also presented. Following the lead of [4, 6, 7, 8], we can obtain local decision rules that maximize the Ali-Silvey distances between the conditional densities at the input of the fusion center

$$D(p(\mathbf{u_F}|H_1),\ p(\mathbf{u_F}|H_0)) = f(E_0[C(\frac{p(\mathbf{u_F}|H_1)}{p(\mathbf{u_F}|H_0)})]), \tag{5}$$

where f is an increasing function; E_0 denotes expectation with respect to p_0; and C is a convex function.

It should be noted that both system designs are obtained under communication constraints given in Equation (1). An example is also presented to illustrate this flexible hybrid decision scheme for the decentralized detection problem. Results show that the system performance of the proposed scheme with lower average communication rate is fairly close to the performance of the centralized system.

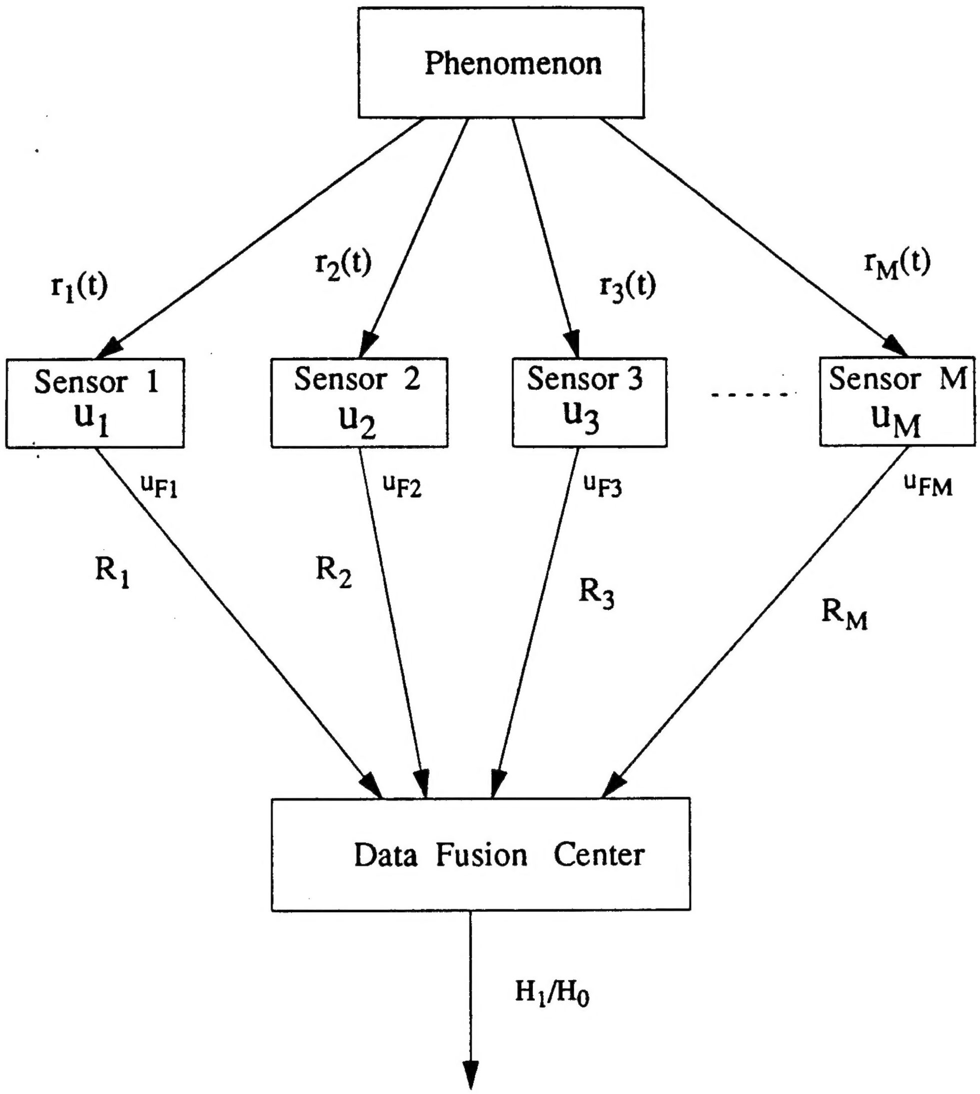

Figure 2: Decentralized Detection System

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